

Statistics

Lecture 15



Feb 19-8:47 AM

Consider the chart below

x	$P(x)$
1	.1
2	.2
3	.3
4	.35
5	.05

1) $P(x=5)$
 $= 1 - [.1 + .2 + .3 + .35] = .05$

2) $P(x \leq 3) = .1 + .2 + .3 = .6$

3) $P(x=2 \text{ or } x=4)$
 $= .2 + .35 = .55$

4) Draw Prob. Dist. histogram

5) $x \rightarrow L1, P(x) \rightarrow L2$
 Use 1-Var stats
 with $L1 \dot{=} L2$
 $\mu = \bar{x} = 3.05$
 $\sigma = \sigma_x = 1.071$
 $n = 1$
 $\mu \approx 3, \sigma = 1$

6) Find σ^2 in Reduced frac.

VARS 5: Statistics f1: σ_x^2

x^2 Math f1: \rightarrow Frac |Enter

$\sigma^2 = \frac{459}{400}$

68% Range $\mu \pm \sigma \Rightarrow 2 \text{ to } 4$
 95% Range $\mu \pm 2\sigma \Rightarrow 1 \text{ to } 5$

Oct 16-12:12 PM

I choose a number between 1 and 250.
 Pay me \$10 and guess what I number
 I picked.

If You guess Correctly, I pay You \$100
 otherwise, I pay nothing.

Expected earning for me Per Play.

Net	P(Net)
10 - 100	$\frac{1}{250}$
10 - 0	$\frac{249}{250}$

L1 } Guess Correctly
 } L2 Wrong Guess

Expected Earning = $\mu = \bar{x}$ \$9.60

Oct 16-12:25 PM

Binomial Prob. dist. SG 16

- n independent events
- Each event has only two outcomes.
 $P(\text{Success}) = p$ $P(\text{Failure}) = q$
 $p + q = 1$, $q = 1 - p$
 p & q remain unchanged for all events
- $x \rightarrow \#$ of Successes
 $n - x \rightarrow \#$ of failures

$P(x) = n C_x \cdot p^x \cdot q^{n-x}$

Binomial
 Prob.
 Dist.

Oct 16-12:30 PM

Consider a binomial Prob. dist. with $n=8$
and $P=.6$.

$$1) q = 1 - P = 1 - .6 = \boxed{.4}$$

$$2) P(x=5) = {}_8C_5 \cdot (.6)^5 \cdot (.4)^3 = \boxed{.279}$$

$$P(x) = {}_n C_x \cdot p^x \cdot q^{n-x}$$

$$3) P(x=4) = {}_8C_4 \cdot (.6)^4 \cdot (.4)^4 = \boxed{.232}$$

$$4) np = 8(.6) = \boxed{4.8}$$

$$5) npq = 8(.6)(.4) = \boxed{1.92}$$

Oct 16-12:35 PM

I flipped a coin 10 times.

$$P(\text{tails}) = .7$$

$$1) n = \boxed{10}$$

$$2) P = \boxed{.7}$$

$$3) q = \boxed{.3}$$

$$4) np = 10(.7) = \boxed{7}$$

$$5) npq = 10(.7)(.3) = \boxed{2.1}$$

$$6) P(\underbrace{\text{exactly } 6}_{x=6} \text{ tails}) = P(x=6)$$

$$= {}_{10}C_6 \cdot (.7)^6 \cdot (.3)^4$$

$$= \boxed{.200}$$

$$7) P(\underbrace{\text{exactly } 8}_{x=8} \text{ tails}) = P(x=8)$$

$$= {}_{10}C_8 \cdot (.7)^8 \cdot (.3)^2$$

$$= \boxed{.233}$$

Oct 16-12:43 PM

You are taking a True-false TEST.

You are making random guesses.

There are 12 questions.

- 1) $n=12$
- 2) $p=.5$
- 3) $q=.5$
- 4) $np=12(.5)$
= $\boxed{6}$
- 5) $npq=12(.5)(.5)$
= $\boxed{3}$
- 6) \sqrt{npq}
= $\sqrt{3} \approx 1.7...$

7) $P(\text{Guess } 8 \text{ questions Correctly})$

$$P(x=8) = {}_{12}C_8 \cdot (.5)^8 \cdot (.5)^4 = \boxed{.121}$$

$${}_n C_x \cdot p^x \cdot q^{n-x}$$

$$P(x=8) = \text{binompdf}(12, .5, 8) = .121$$

using TI Command

```

[2nd] [VARS] ↓ [binompdf] No Menu
Menu 12, .5, 8 [Enter]
n → Trials: 12 [ ] [ ]
P: .5
x: 8 [Paste] [Enter]
    
```

Oct 16-12:50 PM

You are taking an exam.

20 questions, Each question has 4 choices, but only one correct choice per question.

You are making random guesses.

- 1) $n=20$
- 2) $p=\frac{1}{4}=.25$
- 3) $q=\frac{3}{4}=.75$
- 4) $np=20(.25)$
= $\boxed{5}$
- 5) $npq=20(.25)(.75)$
= $\boxed{3.75}$
- 6) \sqrt{npq}
Round to whole #
 $\sqrt{3.75} \approx \boxed{2}$

7) $P(\text{guess } 8 \text{ Questions Correctly})$

$$P(x=8) = \text{binompdf}(20, .25, 8) = \boxed{.061}$$

8) $P(\text{guess at most } 8 \text{ Correctly})$

$$P(x \leq 8) = \text{binomcdf}(20, .25, 8) = \boxed{.959}$$

Oct 16-12:59 PM

Consider a binomial Prob. dist with
 $n=400$ & $P=.8$

1) $q=1-P=.2$ 2) $np=400(.8)=320$ 3) $npq=400(.8)(.2)=64$

4) $\sqrt{npq} = \sqrt{64} = 8$

5) $P(x=325) = \text{binom.pdf}(400, .8, 325) = .042$

6) $P(x \leq 330) = \text{binom.cdf}(400, .8, 330) = .907$
 at most

7) $P(x < 310) = P(x \leq 309)$
 $= \text{binom.cdf}(400, .8, 309) = .096$

Oct 16-1:09 PM

100 newborn babies were randomly selected
 Success is having a boy (girl)

1) $n=100$ 2) $p=.5$ 3) $q=.5$

4) $np=100(.5)=50$ 5) $npq=100(.5)(.5)=25$ 6) $\sqrt{npq} = \sqrt{25} = 5$

7) $P(\text{have at least 40 boys})$
 $x \geq 40$
 $P(x \geq 40) = 1 - P(x \leq 39)$

$= 1 - \text{binom.cdf}(100, .5, 39)$
 $= .982$

Oct 16-1:21 PM

Prob. that you get full recovery from certain surgery is .9.
 50 patients were randomly selected

1) $n = 50$ 2) $p = .9$ 3) $q = 1 - p = .1$

4) $np = 50(.9) = 45$ 5) $npq = 50(.9)(.1) = 4.5$ 6) $\sqrt{npq} = \sqrt{4.5} \approx 2$
 Round to whole #

7) $P(\text{exactly } 40 \text{ have full recovery})$
 $x = 40$
 $P(x = 40) = \text{binompdf}(50, .9, 40) = .015$

8) $P(\text{at most } 48 \text{ have full recovery})$
 $x \leq 48$
 $P(x \leq 48) = \text{binomcdf}(50, .9, 48) = .966$

Oct 16-1:44 PM

9) $P(\text{at least } 45 \text{ have full recovery})$
 $x \geq 45$ Total Prob.

$P(x \geq 45) = 1 - P(x \leq 44)$

~~we don't want~~ we want
 44 45

$= 1 - \text{binomcdf}(50, .9, 44)$
 $= .616$

$P(\text{between } 40 \text{ and } 46, \text{ inclusive have full-recovery})$
 $40 \leq x \leq 46$ $P(40 \leq x \leq 46)$
 $P(x \leq 46)$ $= P(x \leq 46) - P(x \leq 39)$
 $= \text{binomcdf}(50, .9, 46) - \text{binomcdf}(50, .9, 39)$
 $= .740$

Oct 16-1:52 PM

You are taking a multiple-choice exam and making random guesses.

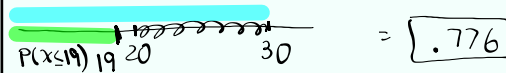
There are 120 questions, 5 choices per question, only one correct choice.

1) $n = 120$ 2) $p = \frac{1}{5} = .2$ 3) $q = \frac{4}{5} = .8$
 4) $np = 120(.2) = 24$ 5) $npq = 120(.2)(.8) = 19.2$ 6) $\sqrt{npq} = \sqrt{19.2} \approx 4.4$
 Round to 1-decimal

7) $P(\text{Correctly guess between } 20 \leq x \leq 30, \text{ inclusive})$

$$P(20 \leq x \leq 30) = \text{binomcdf}(120, .2, 30) - \text{binomcdf}(120, .2, 19)$$

Reduce by 1



Oct 16-2:02 PM

$\mu = np$ Mean
 $\sigma^2 = npq$ Variance
 $\sigma = \sqrt{\sigma^2}$ standard dev.

} Binomial Prob. Dist.

Consider a binomial Prob. dist.

with $n = 250$ & $p = .6$

1) $q = 1 - p = .4$

2) $\mu = np = 250(.6) = 150$

3) $\sigma^2 = npq = 250(.6)(.4) = 60$

4) $\sigma = \sqrt{\sigma^2} = \sqrt{60} \approx 8$

68% Range $\mu \pm \sigma = 150 \pm 8$
 $\Rightarrow 142 \text{ to } 158$

usual Range $\mu \pm 2\sigma = 150 \pm 2(8)$
 $\Rightarrow 134 \text{ to } 166$

Oct 16-2:11 PM

I was bored.

I flipped a Fair Coin 400 times.

Success is landing tails.

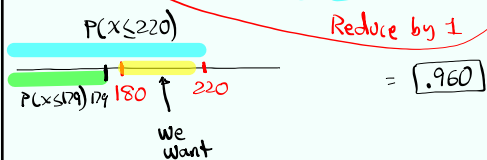
1) $n = 400$ 2) $p = .5$ 3) $q = .5$

4) $\mu = np = 400(.5) = \boxed{200}$ 5) $\sigma^2 = npq = 400(.5)(.5) = \boxed{100}$ 6) $\sigma = \sqrt{\sigma^2} = \sqrt{100} = \boxed{10}$

7) 95% Range $\mu \pm 2\sigma$
 $= 200 \pm 2(10)$
 $= 200 \pm 20 \rightarrow \boxed{180 \text{ to } 220}$

8) $P(\text{lands tails between 180 and 220, inclusive})$

$P(180 \leq X \leq 220) = \text{binomcdf}(400, .5, 220) - \text{binomcdf}(400, .5, 179)$



Oct 16-2:16 PM